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September 1988

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ABSTRACT

In this paper a commonly made assumption in the neoclassical labour supply model is mitigated by introducing hours restrictions into the model. That is, the possibility that an individual is faced with the limited availability of jobs with different, distinct, numbers of hours has been incorporated. Moreover, it has been taken into account that the wage rate may be dependent on the number of working hours. This leads to a nonlinear budget constraint. The results for females suggest not only the existence of hours restrictions, but also of a nonlinear budget constraint. Including both features generates a better fit of the predicted distribution of hours than the standard Tobit model.

1. Introduction

It is well known that in general the predicted distribution of hours in the standard neoclassical labour model fits the actual data on hours very poorly. In this paper we want to tackle this problem. To that end we build a model, which takes its fundaments from Moffitt [1984], who made the budget constraint nonlinear by introducing hours dependent wages, and from Dickens and Lundberg [1985], who dealt with hours restrictions. This model is estimated on Dutch data.

One reason for the bad fit of the hours distribution could be the invalid assumption of a fixed wage rate. Moffitt, among others extended the standard neoclassical labour supply model by making the wage rate endogenous and found significant effects of hours of work on the wage rate, leading to an S-shaped budget constraint. This was in support of the hypothesis put forward by Barzel [1973], namely that the marginal productivity (and thus the marginal wage rate) eventually declines at higher number of working hours. In The Netherlands this argument does not seem very convincing, since most before-tax wage rates are independent of hours of work. Rosen [1976] argued that the wage rate might depend on the number of working hours, due to the fact that there exist different markets for jobs with varying numbers of hours. And there is no reason that the market for full time jobs will clear at the same wage rate as the market for part time jobs. Especially in The Netherlands, where there is a growing interest in part time jobs, mainly by women, this might be an important consideration. Another reason for making the net wage rate dependent on hours of work is the progressive tax system.

Although the model with hours dependent wages fitted the hours distribution better than the standard Tobit model (Moffitt [1984]), the assumption of fixed wage rates seems not to be the only invalid assumption. More important in this respect is the assumption that individuals can freely choose the number of hours they prefer to work. If the diversity of the offered hour packages is large enough, if workers have complete information about job opportunities, and if they are mobile between jobs, they will choose the job with exactly the number of hours they prefer. If workers are not perfectly mobile, for example, they might not be able to work their preferred number of hours. They will choose to work the number of hours that corresponds with the one among the available job offers, yielding highest utility. To our knowledge the first study to estimate a model with hours restrictions on micro data is a paper by Moffitt [1982]. The way we have incorporated hours restrictions is largely based on an article by Dickens and Lundberg. They present a model, in which individuals may face constraints on their number of working hours. Their model is set up as a discrete choice model, in which each worker can choose from a limited number of job offers, with fixed numbers of hours. However they assumed the wage rate to be fixed. In this paper we build a model which incorporates both hours restrictions and hours dependent wages.

The starting point of our analysis is the standard neoclassical model of labour supply, in which individuals maximise a quasi-concave utility function subject to a linear budget constraint. Implicit in this model are the assumptions that labour supply behaviour can be described by a static model, that the wage rate is fixed and that an individual is able to work the number of hours he prefers to work. In this paper we will relax the last two assumptions. That is, the wage rate is made dependent on the number of hours worked, and we incorporate the possibility that an individual is confronted with a limited availability of jobs with different, distinct, numbers of hours. Therefore, it seems appropriate to reformulate the model in terms of a discrete choice problem. By taking into account the availability of jobs with different numbers of hours and hours dependent wages we take a first step in the direction of modelling both the supply side and the demand side of the labour market.

In Section 2 we present a model with both hours restrictions and hours dependent wages. Hours restrictions are incorporated by assuming that employers offer jobs with a fixed number of hours. Workers face the market distribution of these employment opportunities. An individual chooses the number of hours corresponding with that one among the available job offers that yields highest utility. Notice that the individual is still a utility maximizing person, although he maximizes utility on a *subset* of all possible numbers of hours. This subset can be empty, because the number of job offers is a random variable of which zero is one of the possible outcomes. Consequently, the model distinguishes between voluntary and involuntary unemployment. Wages are made endogenous by specifying a wage equation in which the wage rate depends on hours of work and squared hours of work. In Section 3 estimation results for females will be presented and we discuss the improvement of fit of the various model extensions. Section 4 concludes.

2. The model

In this section we will first point out in what way hours restrictions can be incorporated in a standard labour supply model and we discuss the implications of dropping the wage exogeneity assumption. For notational ease subscripts referring to individuals are omitted. In Section 2.1 the likelihood function is derived conditional on the budget constraint. In Section 2.2 the likelihood function will be formulated for the joint wage-hours model.

2.1 Incorporation of hours restrictions

Starting point of the analysis is the following direct utility function (see Hausman [1980], Moffitt [1984]):

$$\log(U(h,y)) = -\log(\gamma-\beta h) - \frac{\beta(h-X\delta-e-\beta y)}{(\gamma-\beta h)} \tag{2.1}$$

where

h := working hours

y := disposable income

X := vector of individual characteristics such as age and family composition

e := random variable, representing unobserved tastes for work

γ, β and δ are parameters

$\gamma>0$, $\beta<0$.

The restrictions $\gamma>0$ and $\beta<0$ are sufficient conditions for monotonicity of the utility function in y.
Maximizing the utility function (2.1) subject to a linear budget constraint yields a linear labour supply function:

$$h = \beta\mu + \gamma w + X\delta + e \tag{2.2}$$

where

μ := nonlabour income

w := wage rate.

The wage equation is specified as

$$w = Z\psi + bh + ch^2 + v \tag{2.3}$$

where

Z := vector of individual characteristics relevant for one's productivity, such as age and education

v := error term

ψ, b, and c are parameters

We assume:

$$\begin{bmatrix} e \\ v \end{bmatrix} \sim N\left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & \rho\sigma_e\sigma_v \\ \rho\sigma_e\sigma_v & \sigma_v^2 \end{bmatrix}\right].$$

If b and c are equal to zero, then the wage rate is fixed and we are dealing with a linear budget constraint. Then the labour supply function given in (2.2) follows from maximizing utility function (2.1). As we have argued before, there are several reasons why the wage rate could depend on hours. This will be the case if there exist different markets for full time and part time jobs, or if there is a progressive tax system. When this wage equation is substituted into the budget constraint

$$y = wh + \mu \tag{2.4}$$

a nonlinear budget constraint results:

$$y = hZ\psi + bh^2 + ch^3 + \mu + hv. \tag{2.5}$$

Maximizing the utility function (2.1) subject to the budget constraint (2.5) yields a nonlinear first order condition in the form of a third order polynomial in h. Estimation of this model would require analytical or numerical solutions to this cubic equation in h. But as will be explained presently we reformulate the model as a discrete choice problem in which utility is compared between a finite number of points of the budget constraint (0, h_1, $h_2,\dots,$ h_m). Therefore it suffices to know the exact specification of the utility function.

We suppose an individual is restricted in his choice of working hours, due to a lack of information or a lack of mobility. If it is assumed that employers offer jobs with fixed numbers of hours, then the worker has to choose from a finite set of jobs, offering fixed numbers of hours. Since working zero hours is always possible, it will be treated as a special case. Let us assume that the market distribution of job offers is the same for all workers, such that the probability of a job offer, which involves $h_\ell(\neq 0)$ working hours is:

$$\Pr(\text{job offer } h=h_\ell) = p_\ell, \qquad \ell=1,\dots,m. \tag{2.6}$$

So we assume that there are m different values of working hours $h_\ell>0$. And there is always the option of working zero hours. Then the labour supply decision becomes a discrete choice out of, let us say, N job offers, drawn from this market distribution of offers, and not working. If the number of job offers, N, approaches infinity, this model becomes a model without hours restrictions, see the Appendix. In that case the worker's behaviour can be described by a discrete choice model in which all possible values of hours are available:

$$h = h_j \quad \text{iff} \quad U(h_j,y_j) > U(h_k,y_k) \quad k=0,\dots,m \quad \text{and } k\neq j \tag{2.7}$$

where U is specified by equations (2.1) and (2.5).
The index k covers the whole range of possible values of hours. However, if individuals face a limited choice of all job offers, then the index k only covers the range of received job offers and zero. One way to model this restricted choice problem is to write down all possible sets of job offers, with corresponding probabilities that an individual will get such a set of offers. Then the probability of observing h_j hours of work is the sum over all sets of the probability that h_j hours is preferred to all job offers in a specific set, times the probability of occurence of that set. Although this way of modelling is appealing for its conceptual simplicity,

it is computationally cumbersome. In this paper we will therefore use a different, equivalent approach. In the Appendix the two methods are written down explicitly. The idea is that an individual is only observed to work h_j hours if he received at least one job offer h_j and if he preferred this job offer to all the other, different offers he received and to not working. The individual is observed as a non-worker if he preferred zero hours to all job offers he received.

Given the values of the two random variables e and v it is possible to construct a set J_j consisting of all job offers preferred less than $h=h_j$:

$$J_j(e,v) = \{ h_\ell : U(h_\ell, y(h_\ell); e,v) < U(h_j, y(h_j); e,v),\ \ell=1,\dots,m\} \qquad (2.8)$$

Notice once more that h=0 is not treated as a job offer. The set $J_j \cup \{h_j\}$ contains all possible job offers an individual could have received if h_j is observed. If this individual would have received an offer that does not belong to $J_j \cup \{h_j\}$, he would have preferred that offer, and he would not have been observed to work h_j hours. Define Q_j as the probability that one draw out of the market distribution of job offers will yield an offer which is less preferred than the chosen h_j, i.e. the offer is in the set J_j:

$$Q_j = \sum_{h_\ell \in J_j} p_\ell \qquad (2.9)$$

Then the probability of observing $h=h_j$ $(h_j \neq 0)$ if N job offers are received can be written as :

$$\begin{aligned} R_j &= (Q_j + p_j)^N - Q_j^N \qquad && \text{if } U(h_j, y(h_j); e,v) > U(0, y(0); e,v) \\ &= 0 && \text{otherwise} \end{aligned} \qquad (2.10)$$

The first line in equation (2.10) describes the probability that the individual only received offers which he preferred less than h_j and that at least one job offer was $h=h_j$. This is equivalent to saying that this individual drew N times a job offer out of $J_j \cup \{h_j\}$ (i.e. $(Q_j + p_j)^N$) but that he did not draw offers *only* out of J_j (Q_j^N). The second line in equation (2.10) says that if zero is preferred to h_j then the probability of observing h_j is zero, since zero is always available.

The probability of observing h=0 when N job offers are received is simply:

$$R_0 = Q_0^N \tag{2.11}$$

Q_0^N is the probability that the N job offers are less preferred than h=0. Recall that so far all formulas are derived, conditional on the values of e and v. Removing the conditioning on v is equivalent to taking into account the endogeneity of the wage rate, and formulating a joint hours-wage probability. This is postponed to the next subsection.

The way to remove the conditioning on the unobserved taste parameter e, is to integrate it out. In doing so, one should remember that the probability R_j is also conditional on the value of e. Then the likelihood of observing $h=h_j$ hours given v, can be written as:

$$L(h=h_j|v) = \int_{-\infty}^{\infty} \varphi(e|v)R_j(e)\ de \tag{2.12}$$

where φ is the normal density function of e given v.
It is clear that J_j, the set of job offers less preferred than h_j, is a step function in e, because only discrete values of hours are considered. Step changes occur at $e=e_{jk}$, i.e. when e takes on such a value that working h_j hours yields the same utility as working h_k hours. See the Appendix for the exact formula of e_{jk}. For values of e between e_{jk} and e_{jk-1} the set $J_j(k)$ remains the same. $J_j(k)$ is defined as J_j for $e_{jk-1}<e< e_{jk}$. Switching from integrals to sums, we can rewrite (2.12) as follows:

$$L(h=h_j|v) = \Pr(\ e< e_{j0}|v)\ R_j(0) + \sum_{k=1}^{m} \Pr(\ e_{jk-1}<e< e_{jk}|v)\ R_j(k)\ + \tag{2.13}$$
$$\Pr(\ e> e_{jm}|v)\ R_j(\text{rest})$$

where

$$R_j(k) = \Pr(\ h=h_j \mid e_{jk-1}<e< e_{jk},\ v).$$

Equation (2.13) can be written explicitly as:

For all $h\neq 0$ and $h\neq h_m$:

$$\begin{aligned} L(h=h_j|v) = \sum_{k=1}^{j-1} & \{\ [\ \Phi(e_{jk}) - \Phi(e_{jk-1})]\ R_j(k)\} + \\ & [\ \Phi(e_{jj+1}) - \Phi(e_{jj-1})]\ R_j(j+1) + \\ \sum_{k=j+2}^{m} & \{\ [\ \Phi(e_{jk}) - \Phi(e_{jk-1})]\ R_j(k)\} + \\ & [\ 1 - \Phi(e_{jm})]\ R_j(\text{rest}) \end{aligned} \tag{2.14}$$

For $h=0$:

$$\begin{aligned} L(h=0|v) = \quad & \Phi(e_{01})\ R_0(1) + \\ \sum_{k=2}^{m} & \{\ [\ \Phi(e_{0k}) - \Phi(e_{0k-1})]\ R_0(k)\} + \\ & [\ 1 - \Phi(e_{0m})]\ R_0(\text{rest}) \end{aligned} \tag{2.15}$$

For $h=h_m$:

$$\begin{aligned} L(h=h_m|v) = \sum_{k=1}^{m-1} & \{\ [\ \Phi(e_{mk}) - \Phi(e_{mk-1})]\ R_m(k)\} + \\ & [\ 1 - \Phi(e_{mm-1})]\ R_m(m) \end{aligned} \tag{2.16}$$

where Φ is the cumulative normal distribution function of e conditional on v and R_j(rest) is defined as in equation (2.10) where e takes a value larger than e_{jm}.

A few remarks have to be made with respect to equations (2.14)-(2.16). First, the values e_{jk} have to be monotonically increasing in k. This will be the case if the budget constraint is linear, but if it is nonlinear this need not be the case. Then they have to be sorted, before the summing in equation (2.14)-(2.16) takes place. Second, the summation is split into two parts by the third term. The reason for this is that e_{jj} is not defined. The ranges $e_{jj-1}<e<e_{jj}$ and $e_{jj}<e<e_{jj+1}$ are combined in $e_{jj-1}<e<e_{jj+1}$.

Until now, we have taken the number of job offers, N, to be fixed. But as mentioned in the introduction, in order to be able to capture the possibility of involuntary unemployment, we make N stochastic. The only difference with the formulas above is that we have to take expectations with respect to the number of job offers:

$$L'(h=h_j|v) = \sum_{N=0}^{Nmax} L(h=h_j|v,N)\cdot p(N) \tag{2.17}$$

where L' is the likelihood when N is a random variable

L is given in equations (2.14)-(2.16)

Nmax is the maximum number of job offers

p is a discrete probability distribution.

Because $L(h=0|v, N=0)=1$ (see formula (2.11)), equation (2.17) turns into

$$L'(h=0|v) = p(0) + \sum_{N=1}^{Nmax} L(h=0|v,N)\cdot p(N) \tag{2.18}$$

for non-workers. So non-working is either explained by the fact that an individual didn't receive any job offers at all (p(0)) or because he preferred not working in the case he received N>0 job offers (second term).

Because $L(h=h_j|v, N=0)=0$ (see formula (2.10)), equation (2.17) can be rewritten as

$$L'(h=h_j|v) = \sum_{N=1}^{Nmax} L(h=h_j|v,N)\cdot p(N) \tag{2.19}$$

for workers.

2.2 Formulation of the joint wage-hours model.

As yet the model has been derived conditional on v. The removal of this conditioning amounts to adding the wage equation to the model and formulating the joint probability of observing h_j hours of work and the corresponding wage rate w. For workers the joint probability can be defined as:

$$L'(h=h_j,w) = L'(h=h_j|v)\ L'(v=w - Z\psi - bh_j - ch_j^2) \tag{2.20}$$

The first term of this probability is given explicitly in equations (2.14)-(2.16). For nonworkers equation (2.20) has to be adapted, since for nonworkers the wage rate is not observed. Therefore, the unobservable wage rate must be integrated out. This results in the following likelihood of observing h=0:

$$L'(h=0) = p(0) + \sum_{N=1}^{Nmax} [\ \Phi(u_{01})\ R_0(1) + \sum_{k=2}^{m} \{ [\ \Phi(u_{0k}) - \Phi(u_{0k-1})]\ R_0(k)\} + [\ 1 - \Phi(u_{0m})]\ R_0(rest)]\cdot p(N) \tag{2.21}$$

where $u_{0k} = e_{0k} + \gamma v$.

As we can see from equation (2.8)-(2.10) the term $R_0(k)$ is calculated for different values of e. Calculation of $R_0(k)$ implies calculation of utility levels, but for non-workers we only have a joint unobserved effect $u=e+\gamma v$, so we cannot calculate utility levels. We are able to circumvent this problem by using the fact that the only thing we need is the ranking of the utilities in different points and not the actual values of the utilities. See the Appendix for further details.

3. Data and estimation results

The model described in Section 2 has been estimated by means of maximum likelihood. The likelihood function is a complex function of the parameters and consequently is maximized by making use of a numerical optimization procedure.

The data come from a labour mobility survey, conducted in The Netherlands in 1985 by order of the Organization of Strategic Labour Market Research (OSA). The sample contains 849 females in families. Sample information is given in Table I.

Table I Sample Characteristics

Variable	Mean (Stand. Dev.)	Minimum	Maximum
hours of work per week	10.6 (15.4)	0	60
	27.3 (12.5)*	2*	60*
net wage rate (guilders per hour)	12.5 (4.5)*	5.8*	39.4*
age	7.1 (9.9)	18	61
educ2 (second level of education)	0.26(0.44)	0	1
educ3 (third level of education)	0.38(0.49)	0	1
educ4 (fourth level of education)	0.02(0.14)	0	1
non-labour income (guild. per week)	714 (302)	0	2693
dummy for children younger than 6	0.26(0.44)	0	1
family size	3.46(1.24)	2	10
dsect (dummy for sector one is educated in; social=0, econ.=2, semi-social, semi-econ.=1)	0.29(0.70)	0	2

number of observations	849
number of working females	331

* := These numbers apply to working females only.

Table II Parameter Estimates (Standard Errors in parentheses)

	No hours restrictions		Hours restrictions	
	linear	nonlinear	linear	nonlinear
	budget constraint		budget constraint	
hours-eq.				
γ (wage)	4.37(0.85)	32.8(52.1)	2.16(0.50)	2.61(0.89)
β (income)	-0.010(0.004)	-0.14(0.23)	-0.004(0.002)	-0.008(0.003)
δ_0(constant)	-48.2(19.9)	179(223)	37.5(9.6)	38.5(13.4)
δ_{10}(age)	-2.16(1.49)	-2.50(10.62)	-1.30(0.72)	-0.88(0.89)
δ_{11}(age-sq.)	0.016(0.019)	-0.06(0.21)	0.011(0.009)	0.005(0.011)
δ_2(fam.size)	-30.7(3.6)	-290(458)	-13.1(2.3)	-19.5(5.5)
δ_3(child.<6)	-19.4(2.8)	-207(328)	-7.89(1.61)	-13.4(4.3)
σ_e	27.4(2.8)	224(348)	11.3(2.1)	13.3(4.2)
wage-eq.				
ψ_0(constant)	-6.90(3.03)	0.23(3.88)	-6.12(2.87)	-0.14(3.67)
ψ_{11}(age)	0.96(0.18)	0.67(0.20)	0.93(0.17)	0.74(0.18)
ψ_{12}(age-sq.)	-0.012(0.002)	-0.009(0.003)	-0.012(0.002)	-0.010(0.002)
ψ_{22}(educ2)	0.044(0.473)	0.24(0.59)	0.058(0.43)	0.086(0.548)
ψ_{23}(educ3)	1.57(0.45)	2.10(0.54)	1.46(0.42)	1.91(0.51)
ψ_{24}(educ4)	4.74(0.73)	6.04(0.77)	4.26(0.67)	5.61(0.76)
ψ_3(dsec)	0.39(0.21)	0.57(0.26)	0.44(0.19)	0.59(0.25)
b (hours)	0(fixed)	-0.004(0.025)	0 (fixed)	-0.078(0.077)
c (hours-sq.)	0(fixed)	-0.002(0.0003)	0 (fixed)	-0.0004(0.001)
σ_v	3.93(0.15)	3.92(0.17)	3.74(0.15)	3.89(0.16)
ρ_{ev}	-0.63(0.09)	-0.35(0.12)	-0.78(0.07)	-0.57(0.12)
job offers				
p1			0.011(0.004)	0.012(0.004)
p2			0.029(0.010)	0.030(0.010)
p3			0.015(0.005)	0.014(0.005)
p4			0.047(0.014)	0.047(0.014)
p5			0.034(0.011)	0.033(0.010)
p6			0.282(0.065)	0.275(0.063)
p7			0.118(0.037)	0.116(0.037)
pn			0.427(0.108)	0.425(0.108)
log lik.	-2183.2	-2106.0	-2041.4	-2034.0

In Table II estimation results are presented. In approximating the budget constraint we have divided the hours range into 4-hours intervals. The first column shows results for the model without hours restrictions and with a wage equation in which the wage is not dependent on hours of work. In principle this is a standard hours-wage model with a linear budget constraint. In the second column the results are presented for the model in which the linearity of the budget constraint is relaxed. The wage rate has become a function of working hours. The last two columns of Table II correspond with the model in which hours restrictions are incorporated. This is implemented by making assumptions about the availability of jobs with a certain number of hours. We have assumed the distribution as shown in Table III.

Table III Offered hours distribution

jobs requiring: ... hours per week	probability of offer	estimated probability in % linear constr.	nonlin.constr.
4,8,12,16	p1	1.14	1.19
20	p2	2.94	2.96
24,28	p3	1.45	1.44
32	p4	4.75	4.65
36	p5	3.37	3.29
40	p6	28.20	27.51
44	p7	11.76	11.65
48,52,...,64	p8	8.30	8.46

The number of job offers is a random variable and is assumed to follow a binomial distribution B(pn,Nmax), where Nmax=10.

Let us now turn to the estimation results presented in Table II. Comparing the likelihoods of column one and two of Table II with each other, and of columns three and four, we can conclude that the hypothesis that the wage rate is independent of hours of work is rejected. In the model with hours restrictions the hours coefficients b and c are insignificant, but their joint effect is significant, although much less substantial than in the model without hours restrictions. In Figure 1 the wage-hours equation is drawn, together with the resulting budget constraint. It follows that wages decline with hours. This may be explained by the progressive tax system in The Netherlands.

Let us now turn to the economic interpretation of the parameters other than those dealing with the shape of the budget line (b, c) or with the hours restrictions (p's). One should bear in mind that in the case of the nonlinear budget constraint the labour supply equation is a cubic equation in h. Therefore we have to be very careful when we compare the values of the estimated parameters in this case with the values in the linear case. In Table II we can see that for all versions of the model, the labour supply curve is forward bending ($\gamma > 0$). Moreover, non-labour income has a negative effect on hours of work, just as family size and a dummy for the presence of children younger than six. Turning to the wage equation, we notice that education has a positive effect on wages and that wages increase with age until about 40 years.

To be able to compare the predicted hours distributions of the different versions, we present some figures. In Figures 2 to 5, hours distributions are drawn for each version of the model. The top panel of Figures 2-5 is the actual hours distribution and the bottom panel is the hours distribution, predicted by the model. From Figure 2 and Figure 3, we observe that the model without hours restrictions does not predict the actual hours distribution very well. This holds both for the model with a linear budget constraint and for the model with a nonlinear budget constraint. In both cases the model misses the peaks at 20, 32 and 40 hours. However, the model with hours restrictions appears to pick up all peaks (see the bottom panels in Figure 4 and 5). The distribution generated by these models is definitely more in line with the actual distribution. Comparing the last four figures we can conclude that after having incorporated hours restrictions not much improvement is gained anymore by making the budget constraint nonlinear. The offered and preferred hours distributions are shown in Figure 6 and 7. In both figures it is striking to see that of all women only between 20 and 30 % prefer not to work, if they are completely free to choose. However, according to the estimated job offer distribution most jobs that are offered require 40 working hours per week. If they have to choose between no job or a 40-hour job, women choose not to work, as can be deduced from Figure 4 and 5.

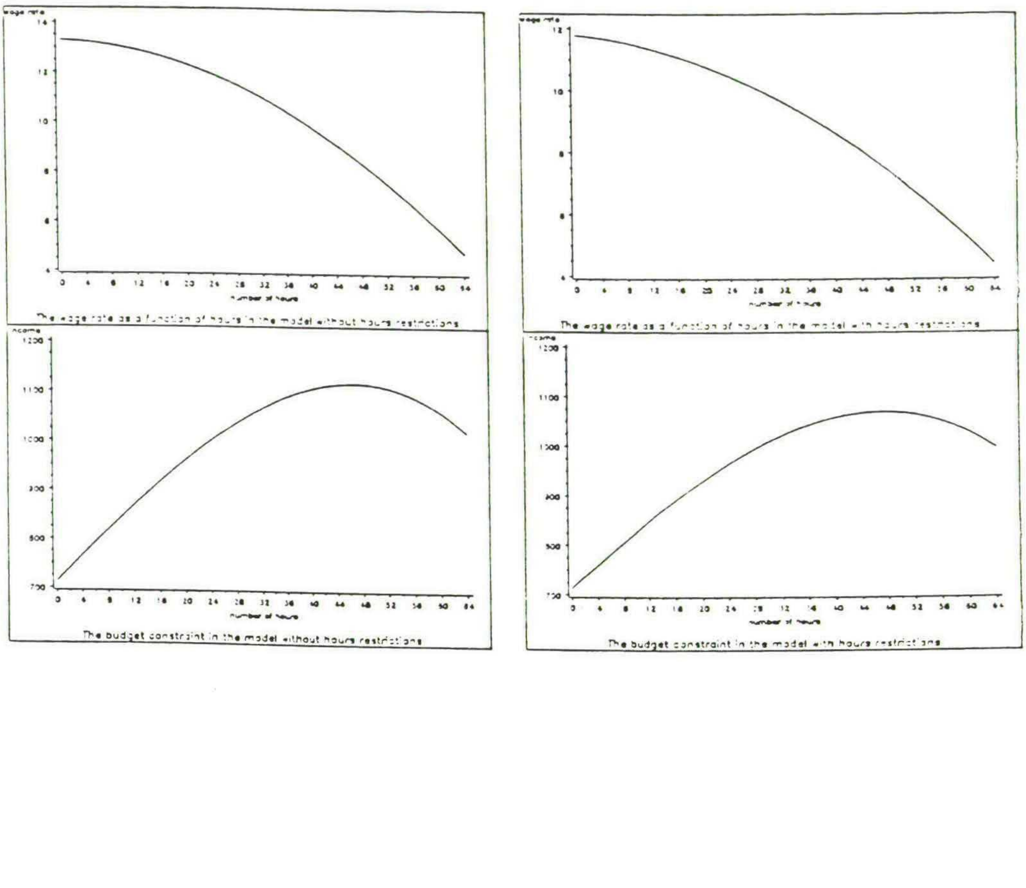

Figure 1 The wage-hours equation and the nonlinear budget constraint.

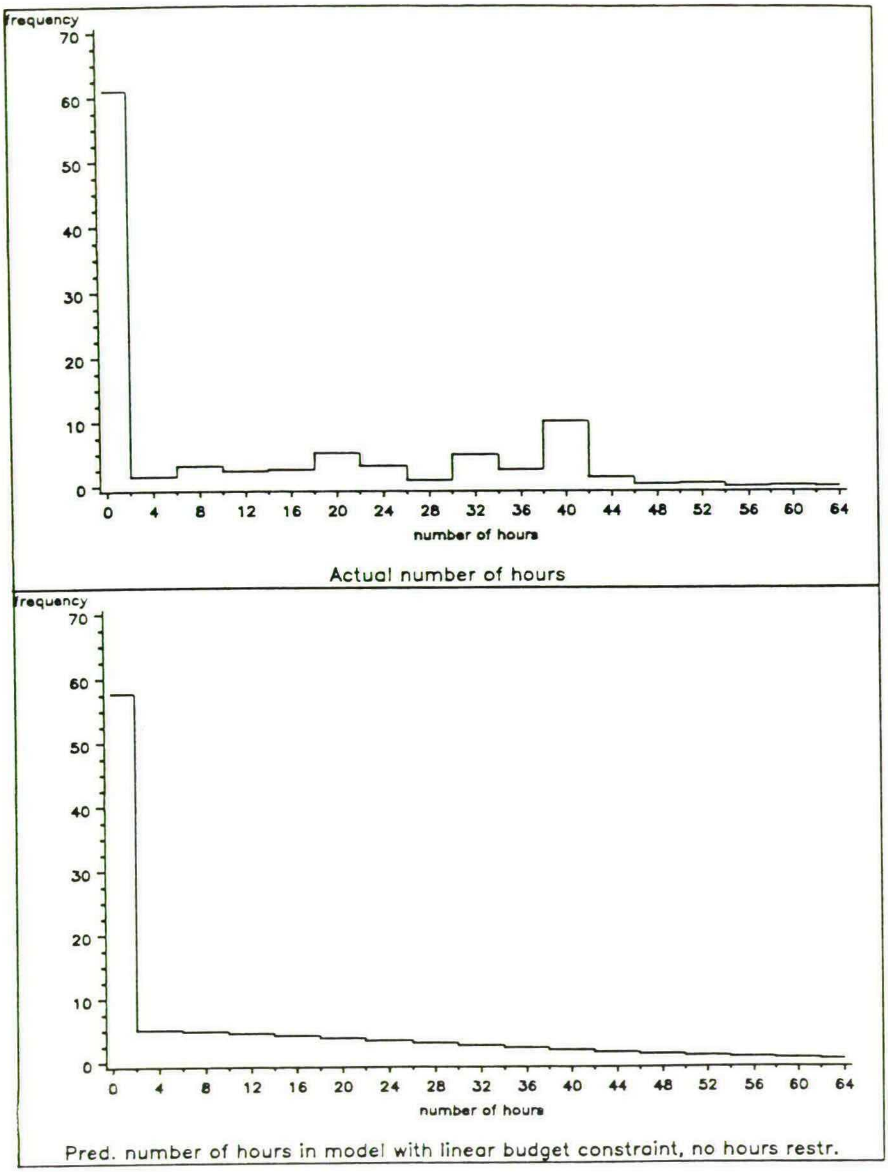

Figure 2 Predicted hours distribution from the model without hours restrictions and with a linear budget constraint in comparison with the actual data.

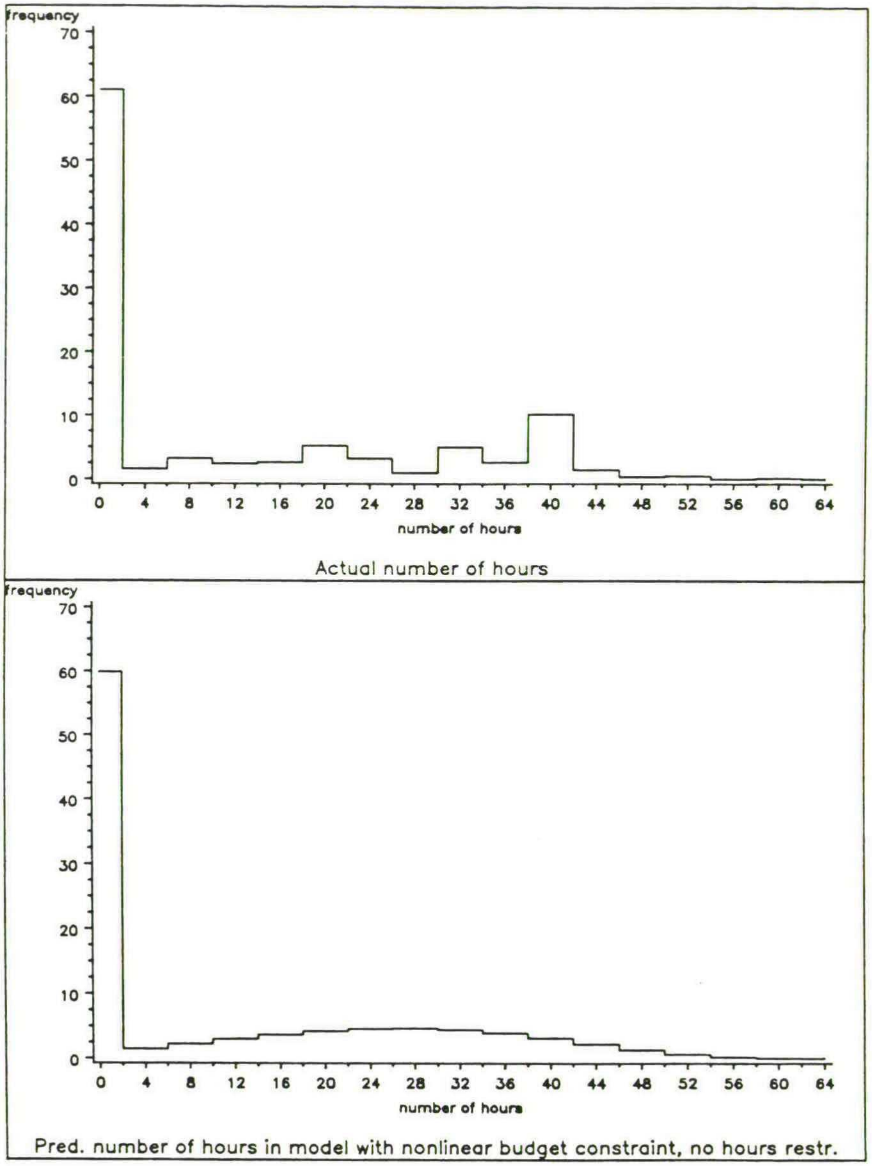

Figure 3 Predicted hours distribution from the model without hours restrictions and with a nonlinear budget constraint in comparison with the actual data.

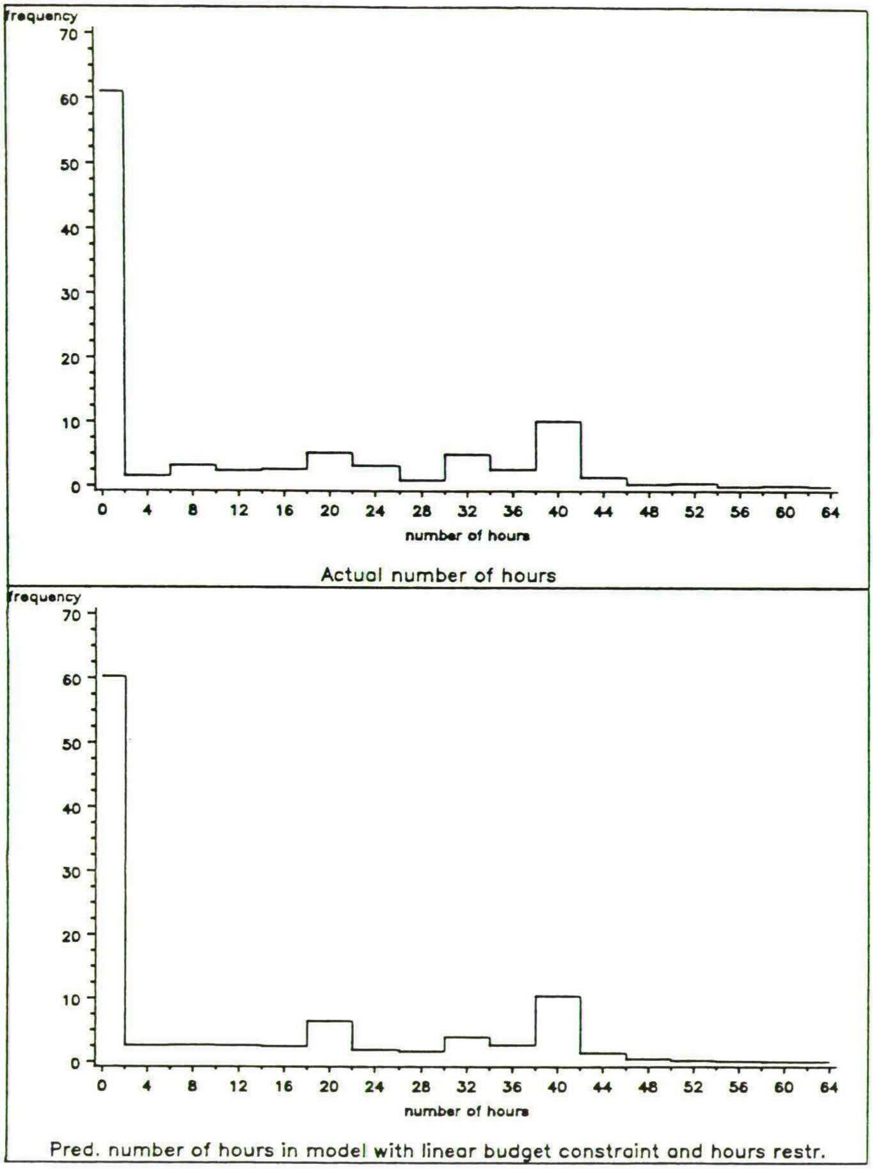

Figure 4 Predicted hours distribution from the model with hours restrictions and with a linear budget constraint in comparison with the actual data.

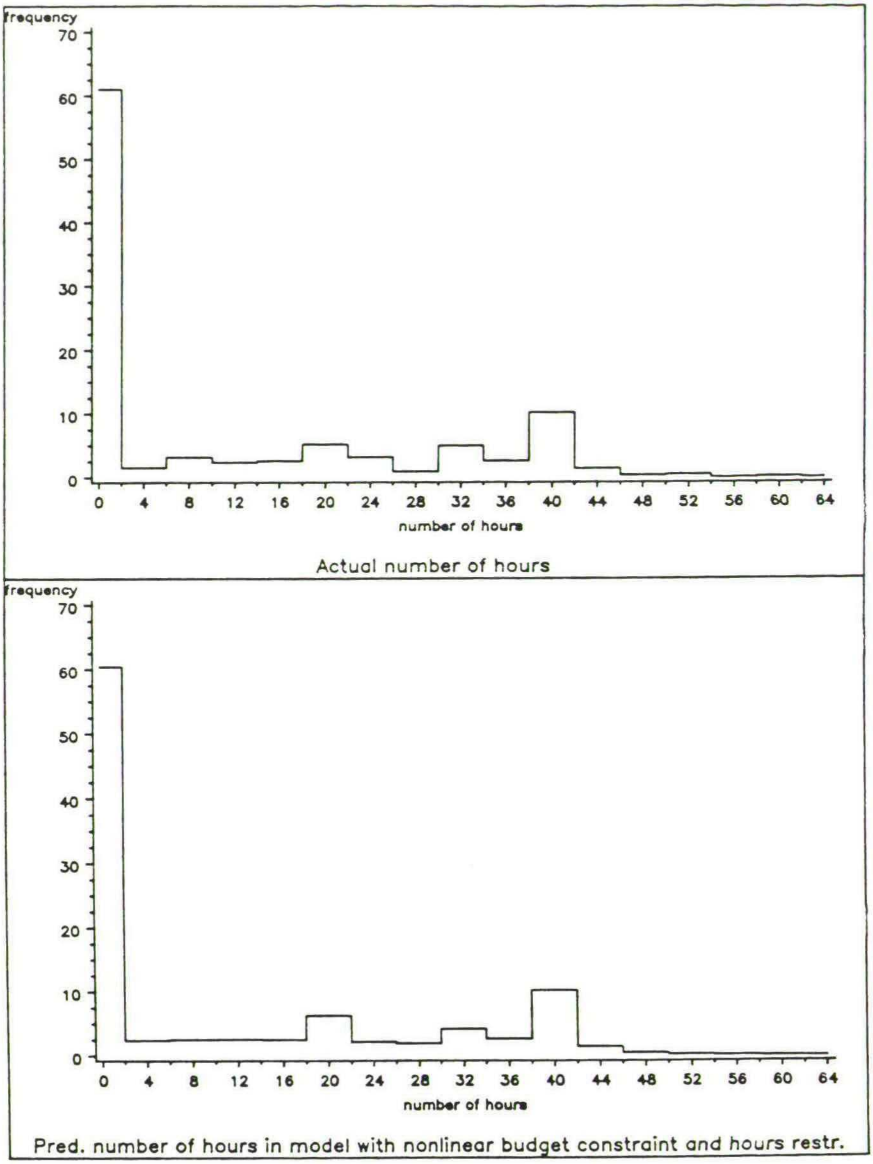

Figure 5 Predicted hours distribution from the model with hours restrictions and with a nonlinear budget constraint in comparison with the actual data.

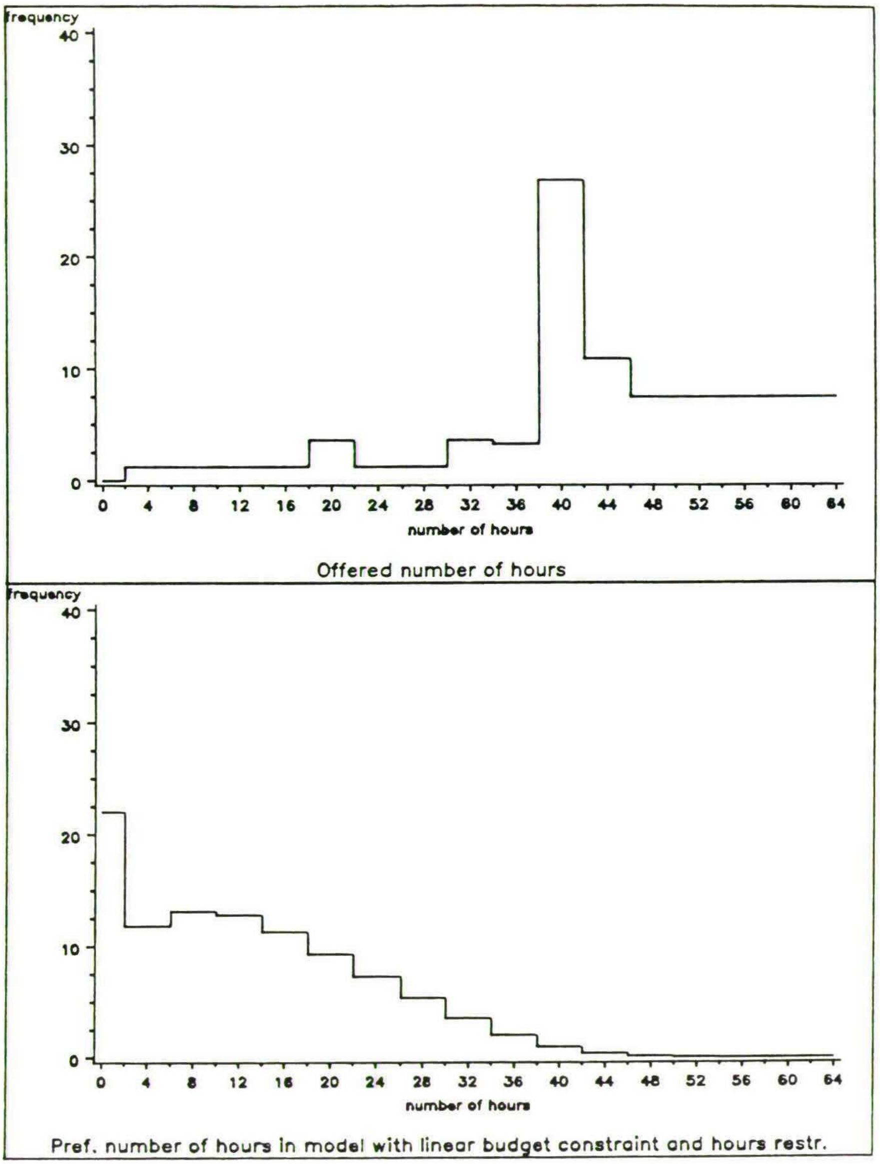

Figure 6 Offered and preferred hours distributions for the model with hours restrictions and with a linear budget constraint.

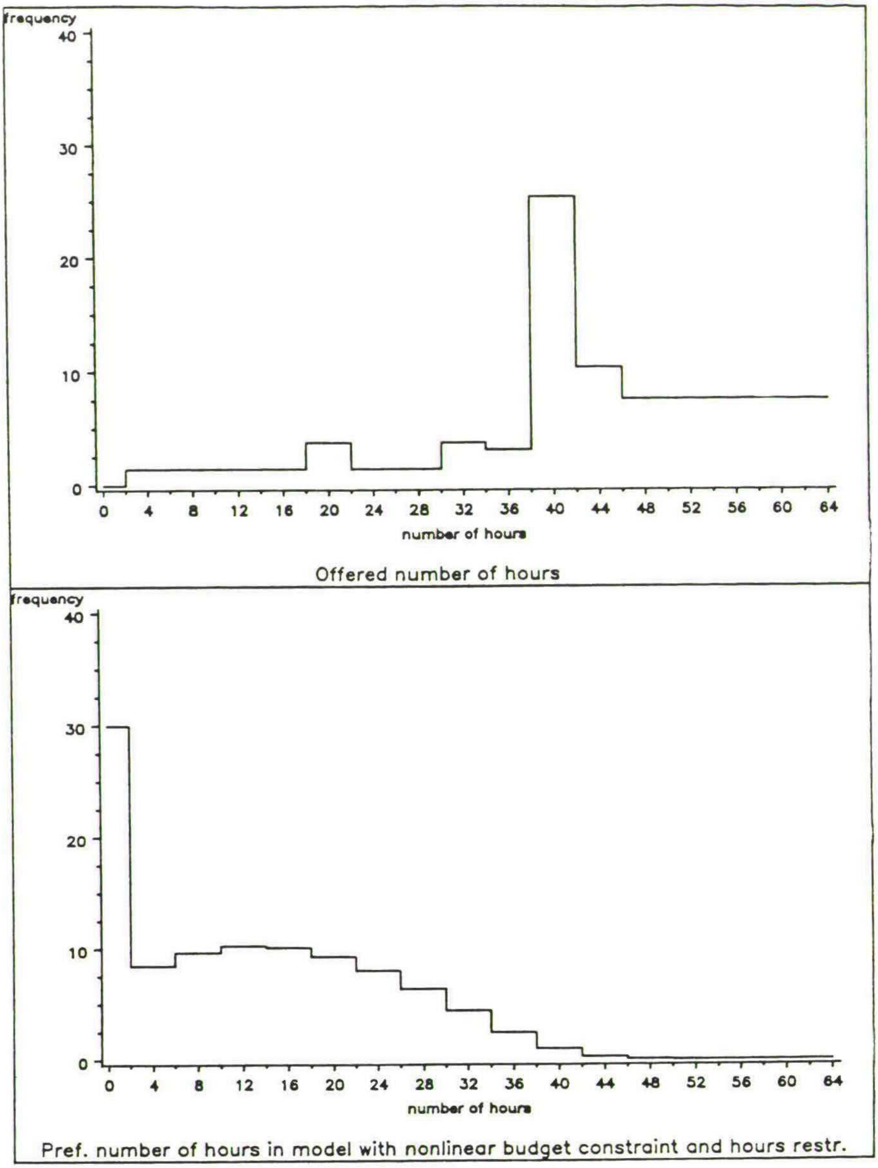

Figure 7 Offered and preferred hours distributions for the model with hours restrictions and with a nonlinear budget constraint.

4. Conclusion

A full simultaneous model of labour supply and wage determination with hours restrictions is estimated. Two main conclusions can be drawn from our analysis. The hypothesis of a linear budget constraint has to be rejected; wages appear to decrease with hours of work. And more importantly, incorporating hours restrictions into the standard labour supply-wage model produces a better approximation of the actual hours distribution.

One of the limitations of this model is that it is static. Although it is obvious that in the future a model of job offers should be dynamic, this model may be a natural first step towards confronting labour supply with the demand side. Another drawback of this study is that the hours restrictions are imposed by a fixed distribution of job offers, common to all individuals. Further research requires a more structural specification in this respect, in order to account for differences in employment opportunities between individuals. Furthermore, tax laws and social security systems could be considered explicitly in describing the budget constraint.

Appendix

Let us first give the exact specification of the e_{jk}'s, i.e. the value of e for which utility in h_j equals utility in h_k. After that we will give an alternative and probably less efficient formulation of our model. Then the entire model we have estimated will be given, and finally we will show that the standard model is a special case of the extended model.

The values e_{jk} follow from equating utility between points h_j and h_k, satisfying the budget constraint.

$$U(h_j,y_j) = U(h_k,y_k) \tag{A.1}$$

Using equation (2.1) and substituting equation (2.5) gives:

$$\log(\gamma-\beta h_j) + \frac{\beta(h_j-X\delta-e_{jk}-\beta y_j)}{(\gamma-\beta h_j)} = \log(\gamma-\beta h_k) + \frac{\beta(h_k-X\delta-e_{jk}-\beta y_k)}{(\gamma-\beta h_k)} \tag{A.2}$$

where

$$y_j = h_j Z\psi + bh_j^2 + ch_j^3 + \mu + h_j v$$

$$y_k = h_k Z\psi + bh_k^2 + ch_k^3 + \mu + h_k v$$

Simple calculations give the solution for e_{jk}:

$$\begin{aligned} e_{jk} = {} & \frac{(\gamma-\beta h_j)(\gamma-\beta h_k)}{\beta^2(h_k-h_j)} \log(\ (\gamma - \beta h_k)/(\gamma - \beta h_j)\ ) + \\ & \frac{(\beta h_j-\gamma)(bh_k^2+ch_k^3) - (\beta h_k-\gamma)(bh_j^2+ch_j^3)}{(h_k-h_j)} + \\ & \gamma/\beta - X\delta - \beta\mu - \gamma Z\psi - \gamma v \\ := {} & u_{jk} - \gamma v \end{aligned} \tag{A.3}$$

In Moffitt [1984] a general rule is derived for which $h=h_j$ is preferred to all other, discrete, number of hours:

$$\max_{k<j} e_{jk} < e < \min_{k>j} e_{jk} \quad \forall\ k \neq j \qquad (A.4)$$

Rule (A.4) expresses a choice $h=h_j$ in an appropriate range of values of the unobserved tastes for work, e. A higher value of e corresponds to a greater taste for work and a lower value of e to a lesser taste for work (see equation (2.2)). Then the rule says that the value of e has to be higher than all those values e_{jk} equating utility between the choice h_j and lower number of working hours ($h_{kl} < h_j$) and has to be lower than all those values e_{jk} equating utility between the choice h_j and higher number of working hours ($h_{kh} > h_j$). This means that the indifference curve for which the choice (h_j, y_j) results needs to be flatter (i.e. higher e) in point (h_j, y_j) than those indifference curves connecting point (h_j, y_j) with points (h_{kl}, y_{kl}) (lower number of working hours) and needs to be steeper (i.e. lower e) in point (h_j, y_j) than those indifference curves connecting point (h_j, y_j) with points (h_{kh}, y_{kh}) (higher number of working hours). This is illustrated in Figure 8.

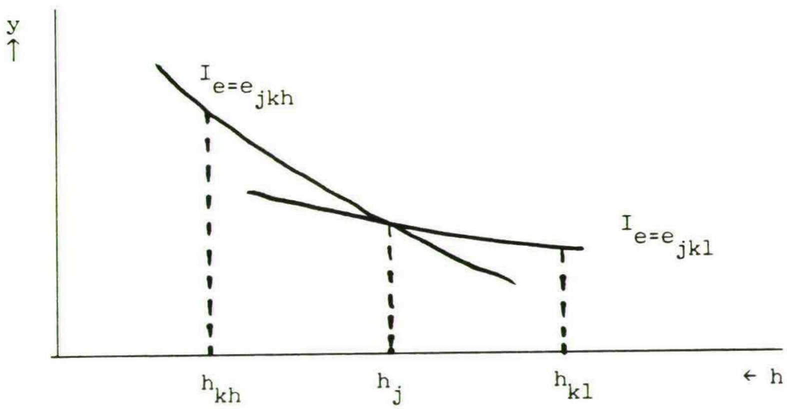

Figure 8 The choice h_j if $\max_{k<j} e_{jk} < e < \min_{k>j} e_{jk}$.

Having this rule for e and e being normally distributed we can define probabiliti s for choice h_j if the choice set consists of all possible numbers of ·orking hours h_k, $k=0,\ldots,m$.

But if there is a limited number of job offers, so that individuals cannot choose freely their optimum number of hours, we have to write down all possible sets of job offers, and their probability of occurring. In writing down the probability of observing h_j hours, we are only interested in the sets with at least one offer h_j. There are $S = \sum_{i=1}^{N-1} \binom{m-1}{i} + 1$ number of these sets, to be called $V_j(s)$. Remember that N is the total number of job offers and m is the number of possible positive distinct hours. Then the probability of observing h_j hours in the set $V_j(s)$ is the probability of occurence of the set $V_j(s)$ times the probability that h_j is preferred to all other numbers of hours in the set $V_j(s)$. The likelihood of observing $h=h_j$ hours, given v, is the sum of the probability of observing h_j in the set $V_j(s)$ over all s:

$$L(h=h_j|v) = \sum_{s=1}^{S} Pr[\max_{k<j} e_{jk} < e < \min_{k>j} e_{jk} | h_k \in V_j(s), v] \; Pr[V_j = V_j(s)] \quad (A.5)$$

We can see that in this way of modelling, determination of the probability of observing h_j is the same as looking for the appropriate range of values of e, for all possible sets containing the offer $h=h_j$.

Instead of this formulation, we could also look for an appropriate set, for all possible values of e. This is nothing else than changing the order of integration. We have now come to the formulation of the model we have estimated, except for the fact that the number of job offers is still fixed:

$$L(h=h_j|v) = \sum_{k=1}^{m} Pr(e_{jk-1} < e < e_{jk} | v) \; Pr(h=h_j | e_{jk-1} < e < e_{jk}, v) \quad (A.6)$$

Remember that

$$Pr(h=h_j | e_{jk-1} < e < e_{jk}, v) = R_j$$

where

$$R_j = (Q_j + p_j)^N - Q_j^N \qquad \text{if } U(h_j, y(h_j); e, v) > U(0, y(0); e, v)$$

$$= 0 \qquad \text{otherwise}$$

Implicit in this probability is the assumption that the individual is still a utility maximizing person, because the set J_j generating the probability Q_j only encloses job offers less preferred than the revealed choice h_j. Since e_{jj} is not defined, and we have to take care of $e<e_{j0}$ and $e>e_{jm}$ as extreme cases, a few adjustments had to be made to obtain equations (2.14)-(2.16). The last term in equation (2.14) $(e>e_{jm})$ takes into account the right tail of the distribution of the unobserved e. For workers the left tail $(e<e_{j0})$ is not included in the summing because not working always belongs to the choice set and therefore working zero hours needs to be less preferred for a worker. So by rule (A.4) the unobserved e has to be greater than e_{j0}.

Furthermore, in the joint hours-wage model we have the difficulty that we do not observe the wage rate for nonworkers. In practice this means that the only term that we can use to define probabilities is $u_{0k} = e_{0k} + \gamma v$ (see also equation (A.3)). But given this joint unobserved effect, we are not able to evaluate utility. For in the utility function we find the expression $e_{0k} - \beta h_k v$, and we only know $e_{0k} + \gamma v$. We have solved this problem by using the fact that we need not know utility at h_k, but we only have to compare it with utility at h=0. We made use of the aforementioned rule:

$$h=h_j \quad \text{iff} \quad \max_{k<j} e_{jk} < e < \min_{k>j} e_{jk} \tag{A.7}$$

Using $u_{jk} = e_{jk} + \gamma v$ this is equivalent with

$$h=h_j \quad \text{iff} \quad \max_{k<j} u_{jk} < e + \gamma v < \min_{k>j} u_{jk} \tag{A.8}$$

For nonworkers this turns into

$$h=0 \quad \text{iff} \quad u < \min_{k>0} u_{0k} \tag{A.9}$$

From rule (A.9) we know that if u is less than u_{0k}, then h_k is not preferred to 0 and therefore belongs to the set J_0:

$$J_0 = \{ h_\ell : u < u_{0\ell}, \ \ell=1,\dots,m \} \tag{A.10}$$

So instead of comparing utilities to determine the set J, we compare the value $u = e + \gamma v$ with values $u_{0\ell}$.

To prove that the model without hours restrictions is a special case of our model if N is fixed, described in Section 2, it is sufficient to show that the probability of observing $h=h_j$ is equal to one for a particular range of e, if the number of job offers tends to infinity. The crucial expression is the probability of observing $h=h_j$ (see equation (2-.10)):

$$R_j = (Q_j + p_j)^N - Q_j^N \tag{A.11}$$

Because Q_j is always smaller than one, R_j tends to one, for N tending to infinity, if $Q_j + p_j$ equals one. The probability $Q_j + p_j$ will only be equal to one for the workers if e falls in a particular range. In all the other cases $Q_j + p_j$ will be less than one and R_j will go to zero. Similarly, there is a range for u such that Q_0 (see (2.12)) will equal one. We know those particular ranges for e and u from equation (A.4) and (A.9). These values for e and u, which are such that working h_j hours is more preferred to working h_k hours, $\forall\ k \neq j$ show up in equations (2.14) and (2.21):

$$[\ \Phi(e_{jj+1}) - \Phi(e_{jj-1})]\ R_j(j+1)$$

and

$$\Phi(u_{01})\ R_0(1)$$

So the model with fixed N converges to the model without hours constraints in Moffitt [1984]. If N is a random variable, it must have a degenerate limiting distribution in infinity in order to attain an equivalent result.

REFERENCES

Ashenfelter, O. 1980. "Unemployment as disequilibrium in a model of aggregate labor supply." *Econometrica* 48(3):547-564.

Barzel, Y. 1973. "The determination of daily hours and wages." *Quarterly Journal of Economics* 87:220-238.

Blundell, R. and I. Walker. 1982. "Modelling the joint determination of household labour supplies and commodity demands." *The Economic Journal* 92:351-364.

Dickens, W. and S. Lundberg. 1985. "Hours restrictions and labor supply." NBER Working Paper No. 1638.

Hausman, J. 1980. "The effect of wages, taxes, and fixed costs on women's labor force participation." *Journal of Public Economics* 14:161-194.

Moffitt, R. 1982. "The Tobit Model, Hours of Work and Institutional Constraints." *The Review of Economics and Statistics* 64:510-515.

Moffitt, R. 1984. "The estimation of a joint wage-hours labor supply model." *Journal of Labor Economics* 2:550-566.

Rosen, H. 1976. "Taxes in a labor supply model with joint wage-hours determination." *Econometrica* 44:485-507.

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